FAR BEYOND

MAT122

Chain Rule

Stony Brook University

Review – Composition of Function

Given: $f(x)=\sqrt{x}$ $g(x)=x^2+1$ find $f(g(x))$

Given: $h(x)=(x^3-1)^{100}$ if $h(x)=f(g(x))$ determine $f(x)$ and $g(x)$

Tip: Pick an *inner* function such that the *outer* function has a SIMPLE derivative.

Differentiating Composed Functions

must take the derivative of BOTH inner and outer functions

if $h(x)$ is in the format $f(g(x))$ then

Chain Rule

$$h'(x) = f'(g(x)) \cdot g'(x)$$

ex: $h(x) = \sqrt{x^2 + 1}$

$\therefore h'(x) =$

$$= \boxed{\frac{x}{\sqrt{x^2 + 1}}}$$

Chain Rule with *u*-Substitution

ex. differentiate $h(x) = (x^3 - 1)^{100}$

define inner function as u: $u = x^3 - 1$

$= \boxed{300x^2 (x^3 - 1)^{99}}$

re-visit: $h(x) = \sqrt{x^2 + 1}$ $u = x^2 + 1$

$= \boxed{\dfrac{x}{\sqrt{x^2 + 1}}}$

Chain Rule

$(outer)' \cdot (inner)'$

$h'(x) = f'(g(x)) \cdot g'(x)$

Chain Rule - examples

ex. find $\frac{dy}{dx}$ of $y = e^{2x}$

Chain Rule

$(outer)' \cdot (inner)'$

$h'(x) = f'(g(x)) \cdot g'(x)$

Product Rule

$y' = f'g + fg'$

$= \boxed{2e^{2x}}$

ex. find y' of $y = (1-x)^2 = u^2$

Chain Rule:

$= \boxed{-2\,(1-x)}$

Product Rule:

$\boxed{= -2(1-x)}$

Practice

Do: find y' of $y = e^{-x}$

$= -e^{-x}$

Do: find y' of $y = e^{kx}$ where k is a constant

$= ke^{kx}$

Chain Rule

$(outer)' \cdot (inner)'$

$h'(x) = f'(g(x)) \cdot g'(x)$

More Chain Rule Examples

ex. find $f'(x)$ when $f(x) = \dfrac{1}{\sqrt[3]{x^2+x+1}}$

Chain Rule

$(outer)' \cdot (inner)'$

$h'(x) = f'(g(x)) \cdot g'(x)$

$$= \boxed{-\frac{2x+1}{3\sqrt[3]{\left(x^2+x+1\right)^4}}}$$

Chain Rule with Product Rule

ex. find y' of $y = (2x+1)^5 x^4$

$$= \boxed{10x^4(2x+1)^4 + 4x^3(2x+1)^5}$$

Chain Rule with Quotient Rule

ex: find derivative of $g(t) = \left(\frac{t-2}{2t+1} \right)^9$

$$= \boxed{\frac{45(t-2)^8}{(2t+1)^{10}}}$$